

# **Interuser Interference Analysis for Direct-Sequence Spread-Spectrum Systems**

## **Part I: Partial-Period Cross-Correlation**

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### **Abstract**

This presentation discusses an analysis approach to evaluate the interuser interference for Direct-Sequence Spread-Spectrum (DSSS) Systems for Space Network (SN) Users. Part I of this analysis shows that the correlation property of pseudo noise (PN) sequences is the critical factor which determines the interuser interference performance of the DSSS system. For non-standard DSSS systems in which PN sequence's period is much larger than one data symbol duration, it is the partial-period cross-correlation that determines the system performance.

This study reveals through an example that a well-designed PN sequence set (e.g. Gold Sequence, in which the cross-correlation for a whole-period is well controlled) may have non-controlled partial-period cross-correlation which could cause severe interuser interference for a DSSS system. Since the analytical derivation of performance metric (bit error rate or signal-to-noise ratio) based on partial-period cross-correlation is prohibitive, the performance degradation due to partial-period cross-correlation will be evaluated using simulation in Part II of this analysis in the future.



# Interuser Interference Analysis for Direct-Sequence Spread-Spectrum Systems Part I: Partial-Period Cross-Correlation

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- **Purpose**
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# Purpose



- To develop an analysis approach to evaluate the interuser interference for Direct-Sequence Spread-Spectrum (DSSS) Systems for Space Network (SN) Users



# Problem Definition



- **Pseudo-random (PN) sequences used for DSSS multiple access systems generally have good cross and auto correlation properties on their whole period. These properties adequately apply to the so called standard DSSS systems in which a data symbol is spread by a whole PN sequence period**
  - The performance of such systems depends on the aperiodic correlation properties [1]
- **In many applications, it's necessary to employ PN sequences whose period is much larger than one data symbol duration. In such cases, it is the partial-period cross-correlation that determines the system performance. When the data detection integrates over a symbol duration which only spans a portion of a PN sequence, some of the well-designed correlation properties of that PN sequence is lost. This could result in the degraded interference immunity.**
- **Since DSSS systems in SN could fall into the above category, we will investigate the partial-period correlation properties of those PN sequences used by SN for the purpose of radio frequency compatibility (RFC) analysis**



# Signal Models



Transmitted desired signal

$$s_0(t) = \sqrt{2P_0} b_0(t) c_0(t) \cos(\omega_c t + \phi_0)$$

Transmitted interfering signal

$$s_k(t) = \sqrt{2P_k} b_k(t) c_k(t) \cos(\omega_c t + \phi_k)$$

Received signal

$$r(t) = s_0(t - \tau_0) + \sum_{k=1}^{N-1} s_k(t - \tau_k) + n(t)$$

Received signal with single interfering user k

with equal symbol rate and equal chip rate

(without loss of generality, assume  $\tau_0 = 0$ ,  $\phi_0 = 0$ )

$$r(t) = \sqrt{2P_0} b_0(t) c_0(t) \cos(\omega_c t) + \sqrt{2P_k} b_k(t - \tau_k) c_k(t - \tau_k) \cos(\omega_c t - \tau_k + \phi_k) + n(t)$$



# Test Statistics



- The test statistics of symbol  $i$  of desired user 0 at output of matched filter / correlator receiver (after despreading and down-conversion assuming established synchronization of user 0)

$$\begin{aligned}\xi_i &= \int_0^T r(t) c_0(t) \cos(\omega_c t) dt \\ &= \sqrt{P_0 / 2} b_{0,i} T \\ &\quad + \sqrt{P_k / 2} (b_{k,i-1} R_{k,0}(\tau_k) + b_{k,i} \hat{R}_{k,0}(\tau_k)) \cos \phi_k \\ &\quad + \int_0^T n(t) c_0(t) \cos(\omega_c t) dt\end{aligned}$$

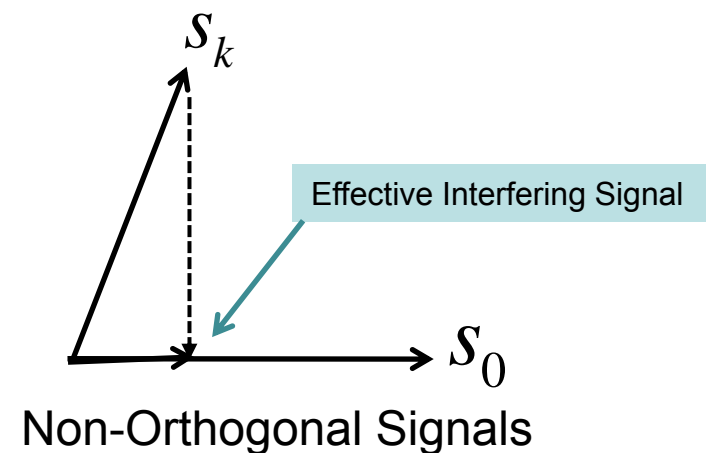
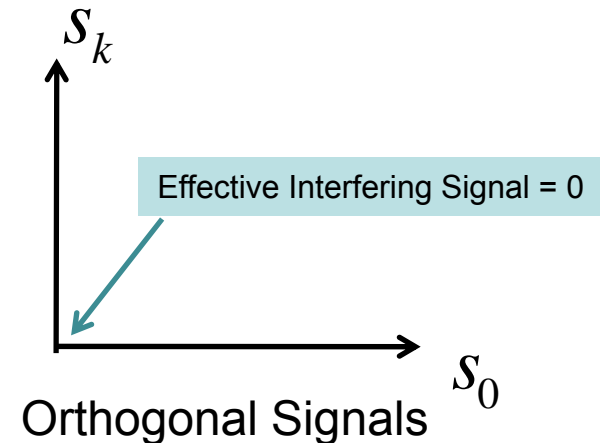
where  $T$  is the duration of a data symbol,

$R_{k,0}(\tau_k)$  and  $\hat{R}_{k,0}(\tau_k)$  are partial cross-correlations of two PN sequences.



# Cross-Correlation

- The interference (second item) in the test statistics on page #6 is of some form of cross-correlation between two PN codes.
- Cross-correlation is the inner product of two signal vectors, which is the measure of agreement between two PN codes. The effective interference can be considered as the projection from the interfering signal into the desired signal.
- In CDMA multiple users occupy the same RF bandwidth and transmit simultaneously. When the user codes are orthogonal (cross-correlation goes to zero), there is no interference between users after despreading.
- In practice, the PN codes are not perfectly orthogonal; hence the cross-correlation between user codes introduces performance degradation due to the non-zero effective interfering signal after despreading.







# Cross-Correlation



- Only for a special case (usually non-practical) in which the interfering user  $k$  is synchronized with desired user 0,

$$\xi_i = \sqrt{P_0/2}b_{0,i}T + \sqrt{P_k/2}b_{k,i}R_{k,0} + \int_0^T n(t)c_0(t)\cos(\omega_c t)dt$$

where

$$R_{k,0}(\tau_k) = \int_0^T c_k(t)c_0(t)dt \text{ is the cross-correlation of two spreading sequences.}$$

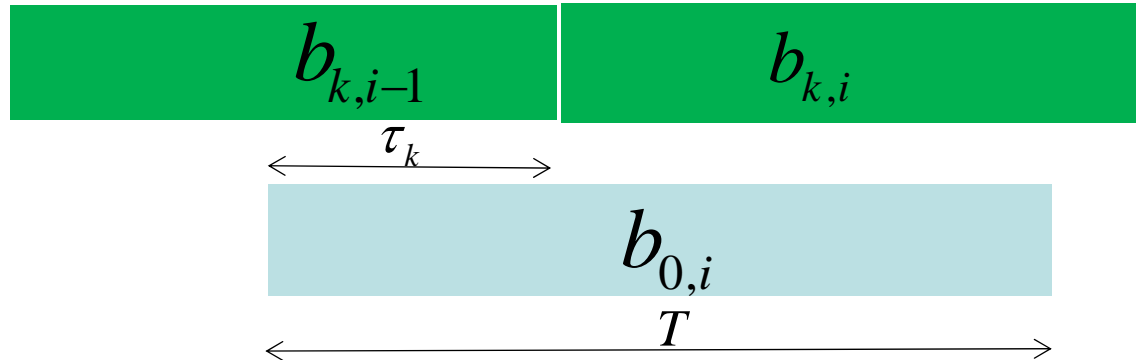
$$\text{and } SNR \square \frac{\sqrt{P_0/2T}}{\sqrt{Var(\xi_i)}} = \sqrt{\frac{2E}{N_0 + \frac{2P_k R_{k,0}^2}{T}}}$$



# Partial Cross-Correlation



- Due to channel asynchronization



$$R_{k,0}(\tau_k) = \int_0^{\tau_k} c_k(t - \tau_k) c_0(t) dt,$$

$$\hat{R}_{k,0}(\tau_k) = \int_{\tau_k}^T c_k(t - \tau_k) c_0(t) dt$$



# Aperiodical Cross-Correlation



- The discrete aperiodic cross-correlation function is defined by

$$C_{k,0}(l) = \begin{cases} \sum_{j=0}^{N-1-l} c_j^{(k)} c_{j+l}^{(0)}, & 0 \leq l \leq N-1 \\ \sum_{j=0}^{N-1+l} c_{j-l}^{(k)} c_j^{(0)}, & 1-N \leq l \leq 0 \\ 0, & |l| \geq N. \end{cases}$$

where  $N = T / T_c$ ,

$T_c$  is the chip duration,

$l$  is the delay index in term of chips;

For  $0 \leq lT_c \leq \tau_k \leq (l+1)T_c \leq T$ ,

$$R_{k,0}(\tau_k) = C_{k,0}(l-N)T_c + [C_{k,0}(l+1-N) - C_{k,0}(l-N)](\tau_k - lT_c),$$

$$\hat{R}_{k,0}(\tau_k) = C_{k,0}(l)T_c + [C_{k,0}(l+1) - C_{k,0}(l)](\tau_k - lT_c).$$



## Variance of Test Statistics



- The conditional variance with respect to the mutually independent random variables (  $\phi_k, \tau_k, b_{k,i-1}, b_{k,i}$  ) is

$$\text{Var}(\xi_i) = \frac{P_k T^2}{12N^3} r_{k,0} + \frac{N_0 T}{4}$$

where

$$\begin{aligned} r_{k,0} = & \sum_{l=0}^{N-1} [C_{k,0}^2(l-N) + C_{k,0}(l-N)C_{k,0}(l-N+1) \\ & + C_{k,0}^2(l-N+1) + C_{k,0}^2(l) + C_{k,0}(l)C_{k,0}(l+1) \\ & + C_{k,0}^2(l+1)]. \end{aligned}$$



# Signal-to-Noise Ratio



- The signal-to-noise ratio is defined by

$$SNR \square \frac{\sqrt{P_0 / 2T}}{\sqrt{Var(\xi_i)}}$$
$$= \sqrt{\frac{2E}{N_0 + \frac{P_k [2\mu_{k,0}(0) + \mu_{k,0}(1)]T}{3N^3}}}$$

where

$$\mu_{k,0}(0) = \sum_{l=0}^{N-1} [C_{k,0}^2(l-N+1) + C_{k,0}^2(l+1)],$$

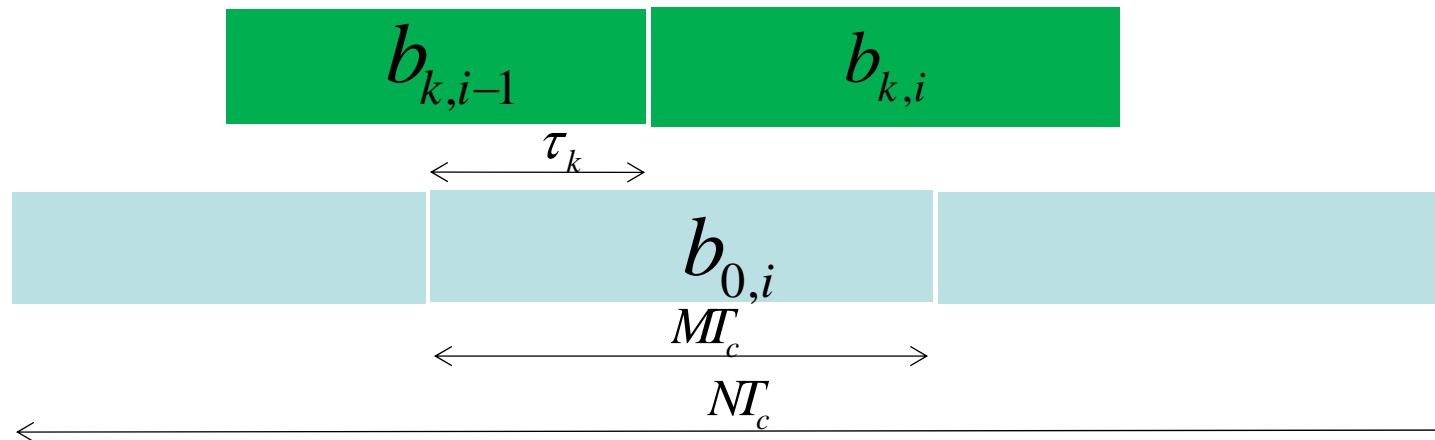
$$\mu_{k,0}(1) = \sum_{l=0}^{N-1} [C_{k,0}(l-N)C_{k,0}(l-N+1) + C_{k,0}(l)C_{k,0}(l+1)].$$



# Partial-Period Cross-Correlation



- For most practical DSSS systems, the symbol period ( $T=MT_c$ ) is less than the PN sequence period ( $NT_c$ ) for  $M < N$ , the correlation for data detection is only over the partial period (P-P)  $M$  of the PN sequence and is defined as



$$R_{k,0}^{P-P}(\tau_k) = \int_{lT_c}^{lT_c + \tau_k} c_k(t - \tau_k) c_0(t) dt,$$

$$\hat{R}_{k,0}^{P-P}(\tau_k) = \int_{lT_c + \tau_k}^{lT_c + T} c_k(t - \tau_k) c_0(t) dt$$



## PN Codes for Space Network [2]



- **PN codes are used to produce ranging services, CDMA services, and satisfy PFD restrictions.**
- **The SN assigns PN codes for NASA, ESA and NASDA for interoperability. Four link Libraries of the 85 code/ code pair assignments are designated for each agency.**
- **Two types of PN codes generated by shift-registers are utilized in code libraries: Gold Codes, which contain fixed feedback tap locations and user specific initial conditions; and Maximal Length Codes (M Codes), which contain user-specific feedback tap locations and an all 1's initial condition.**
- **Forward Command Link (I channel) and Return Link Mode 2 code libraries consist of Gold Codes while Forward Range Channel (Q channel) and Return link Modes 1 and 3 consist of M Codes.**
- **There is a special code library for Shuttle S-band Command Link which has four Maximal Length Codes of length 1023.**
- **All the codes from a given set are generated by essentially the same circuit which permit interoperability and avoid mutual interference by the code selection criteria.**



## Example (ATV vs. HTV)



- **The ATV docking and HTV berthing to the ISS is used as an example to illustrate the analysis approach**
- **ATV forward command link parameters selected [3]:**
  - Assigned PN Code: ESA PN code 25
  - Chip rate: ~ 3 Mcps (3.078 Mcps)
  - Data rate : 1 kbps (rate 1/2 Convolutional Coded to 2 ksps);
- **HTV forward command link parameters selected [4]:**
  - Assigned PN Code: NASDA PN code 8
  - Chip rate: ~ 3 Mcps (3.078 Mcps)
  - Data rate: 250 bps (uncoded)

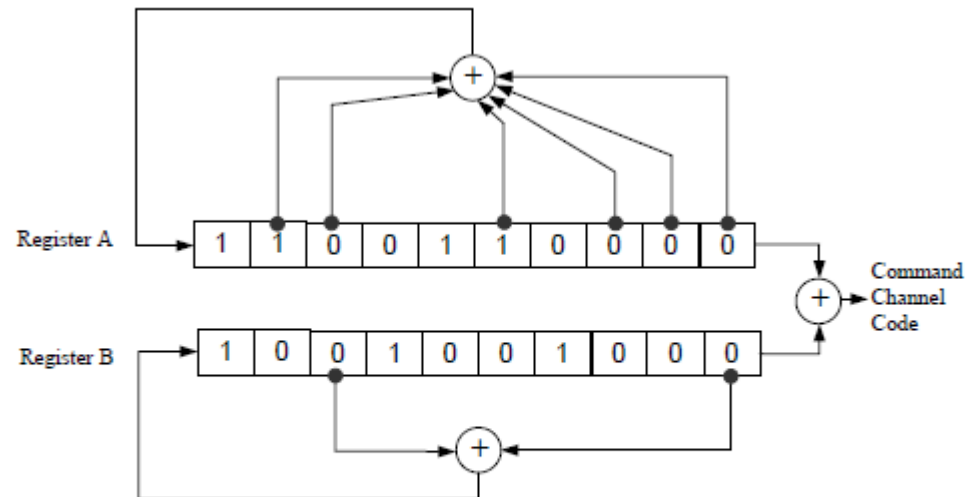




# ATV Forward Command Link PN Code [2]



- **ESA PN Code 25**
- **Initial Conditions: 110011000**



ESA PN Code 25

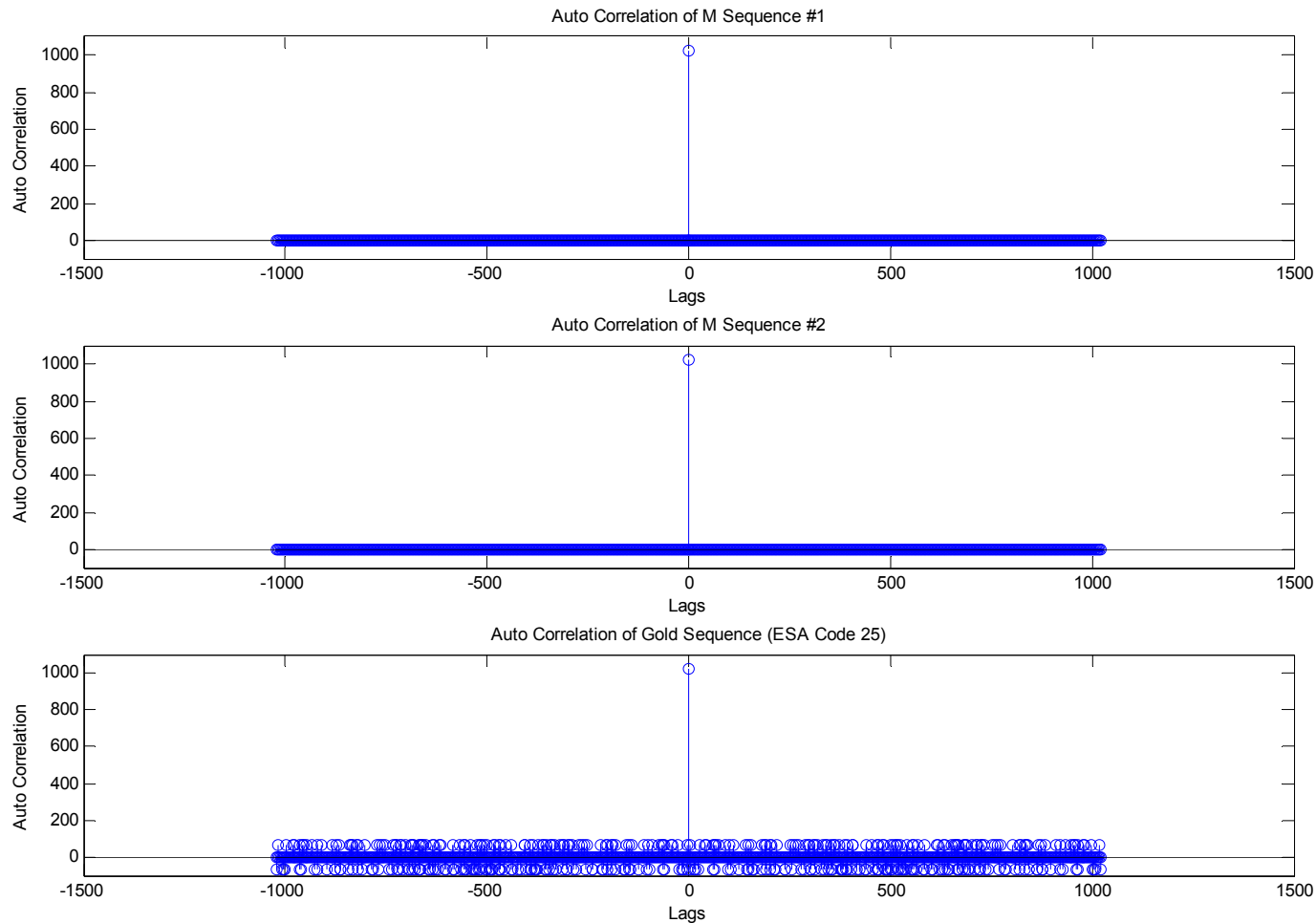
Figure 3-3. ATV Command Channel PN Code Generator



# ATV Forward Command Link PN Code (autocorrelation)



- Simulation shows good autocorrelation property.





# HTV Forward Command Link PN Code [2]



- NASDA PN Code 8
- Initial Conditions: 1000010000

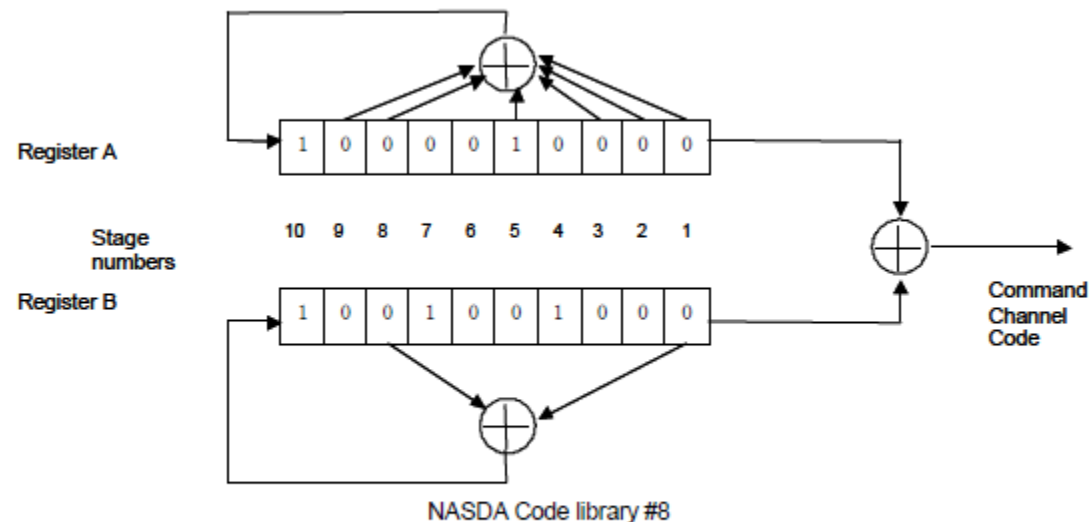


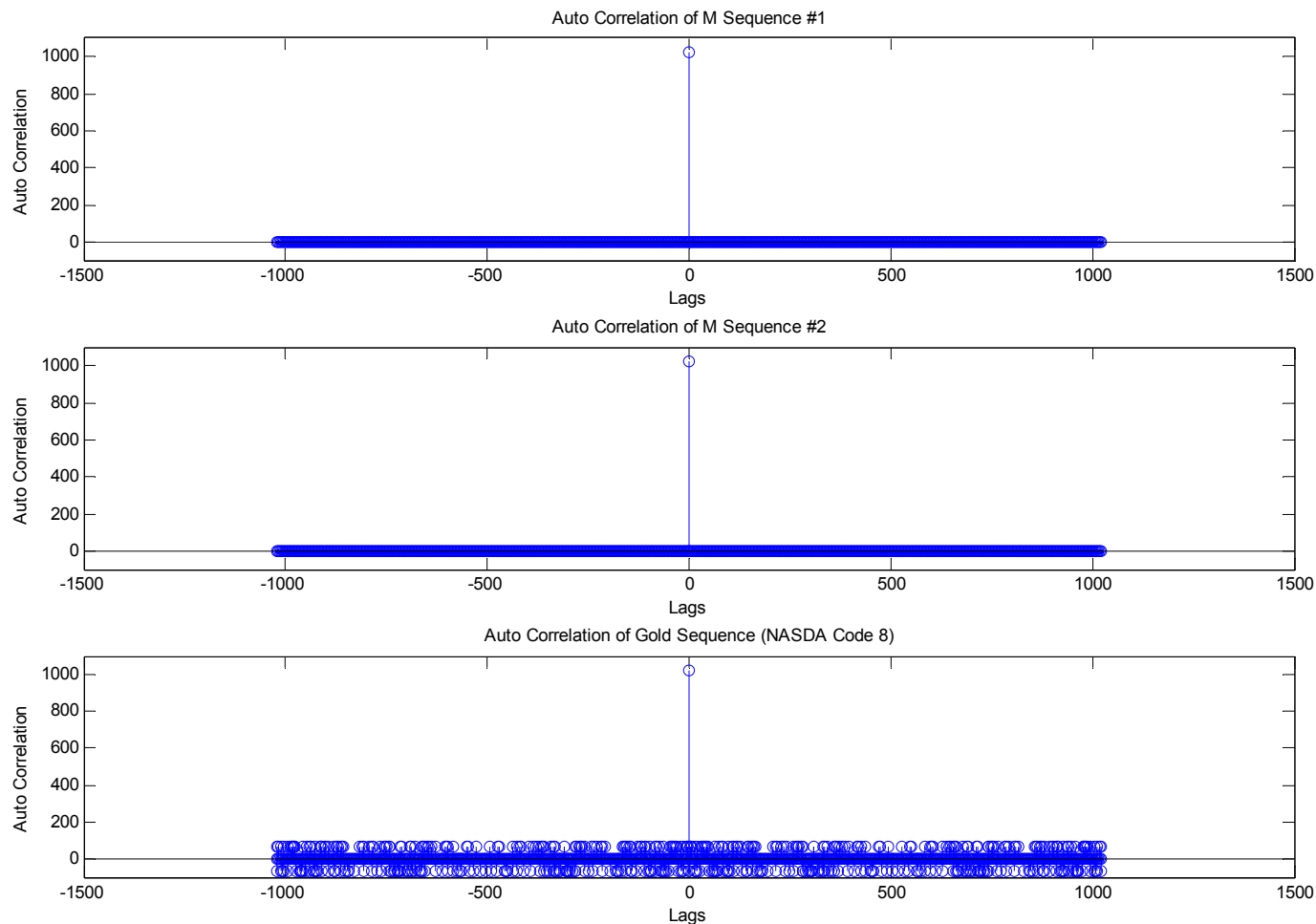
Figure 4-12. SN to HTV command channel short PN code generation functional configuration



# HTV Forward Command Link PN Code (autocorrelation)



- Simulation shows good autocorrelation property.





## Three Value Cross-Correlation of Gold Codes



- Cross-Correlation of Gold Codes only takes three values:

$$R = \begin{cases} -1 \\ -(2^{(L+1)/2} + 1) \text{ for } L \text{ is odd;} \\ (2^{(L+1)/2} - 1) \end{cases}$$

$$R = \begin{cases} -1 \\ -(2^{(L+2)/2} + 1) \text{ for } L \text{ is even } (L=2 \bmod 4) ; \\ (2^{(L+2)/2} - 1) \end{cases}$$

where L is the number of stages of the shift register.

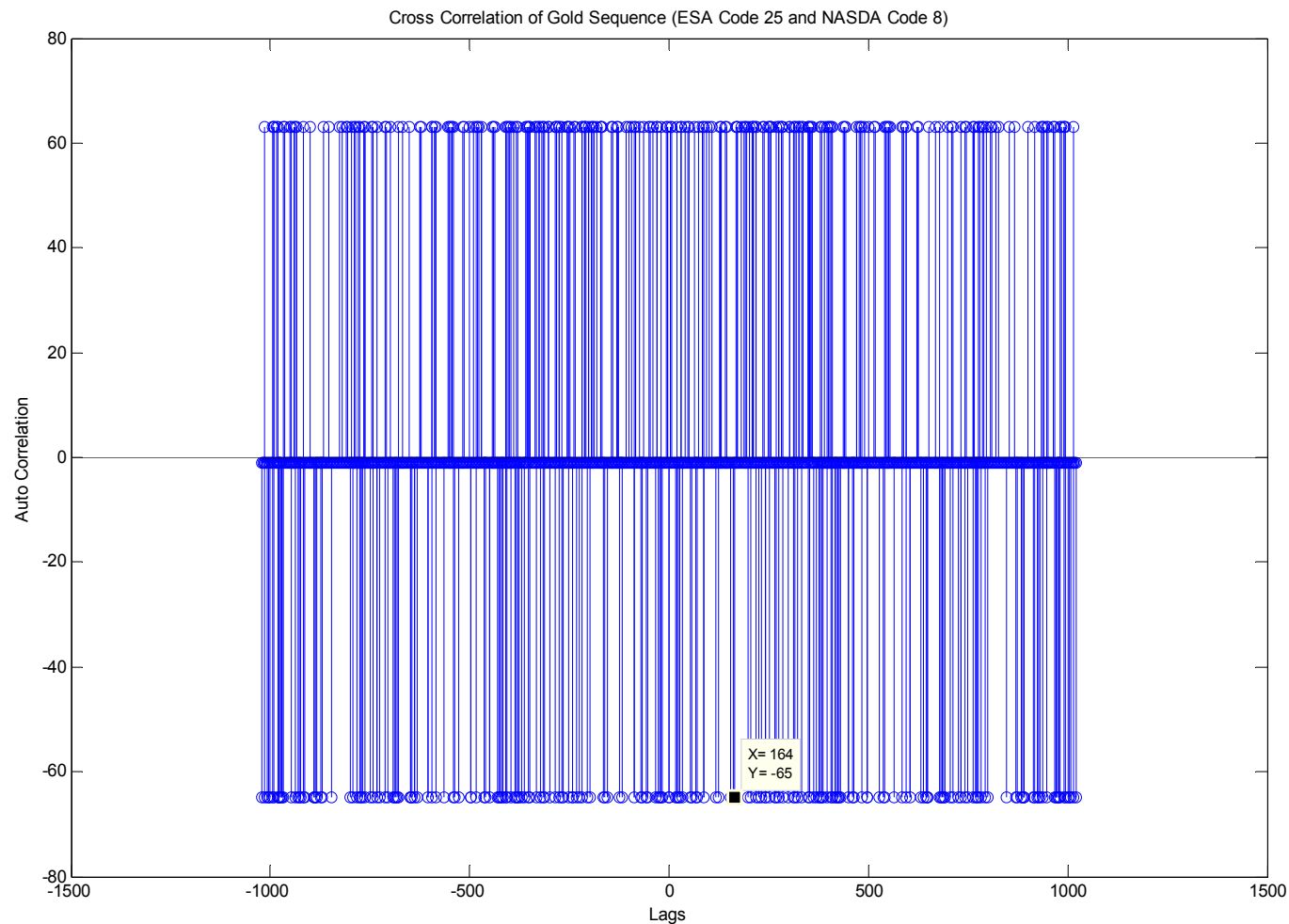
Example: L=10, R=63, -1, -65.



# Cross-Correlation (ATV Code vs. HTV Code)

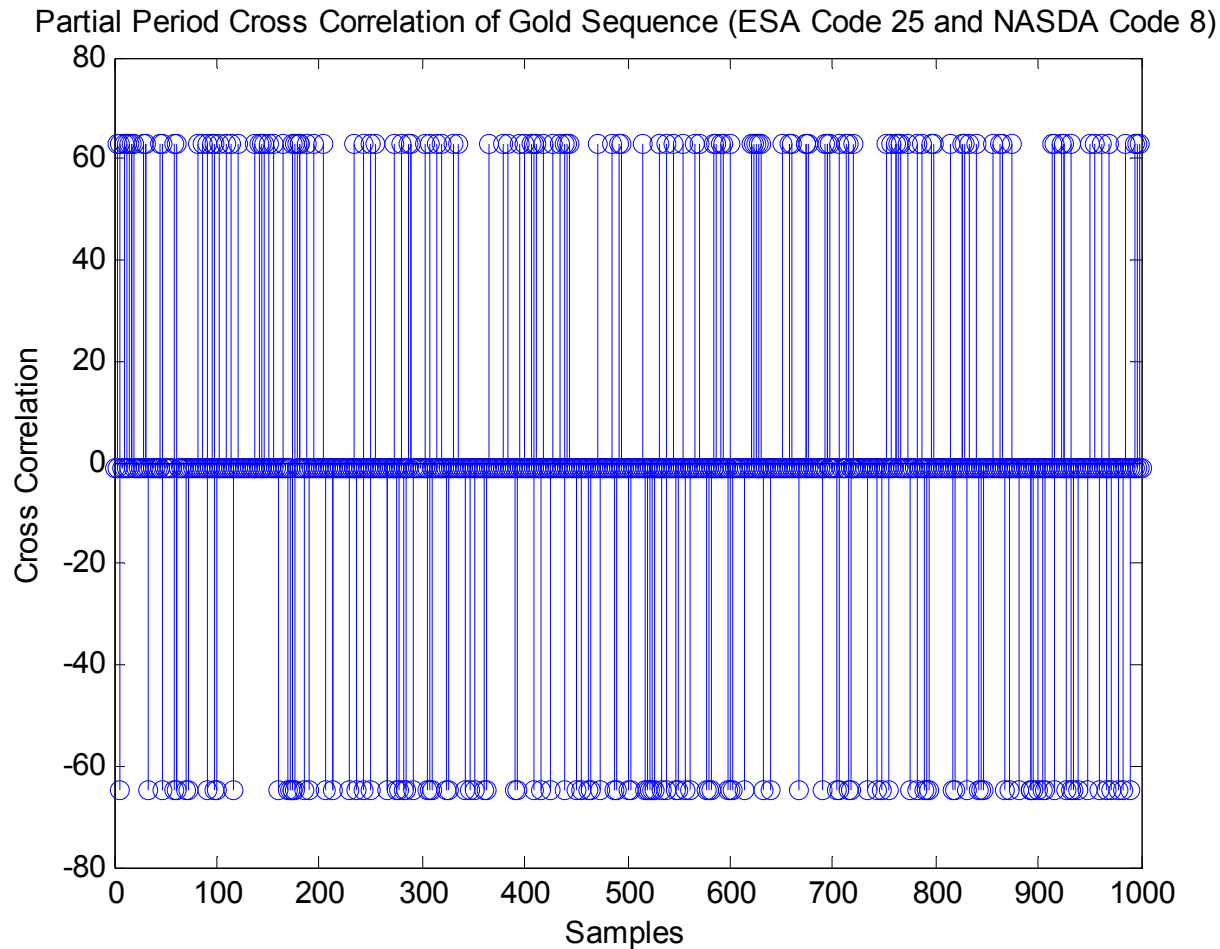


- Simulation shows well-controlled cross-correlation property matching with the theoretical results.





## Partial-Period Cross-Correlation of ATV Code and HTV Code (Partial-Period M=1023 Chips)

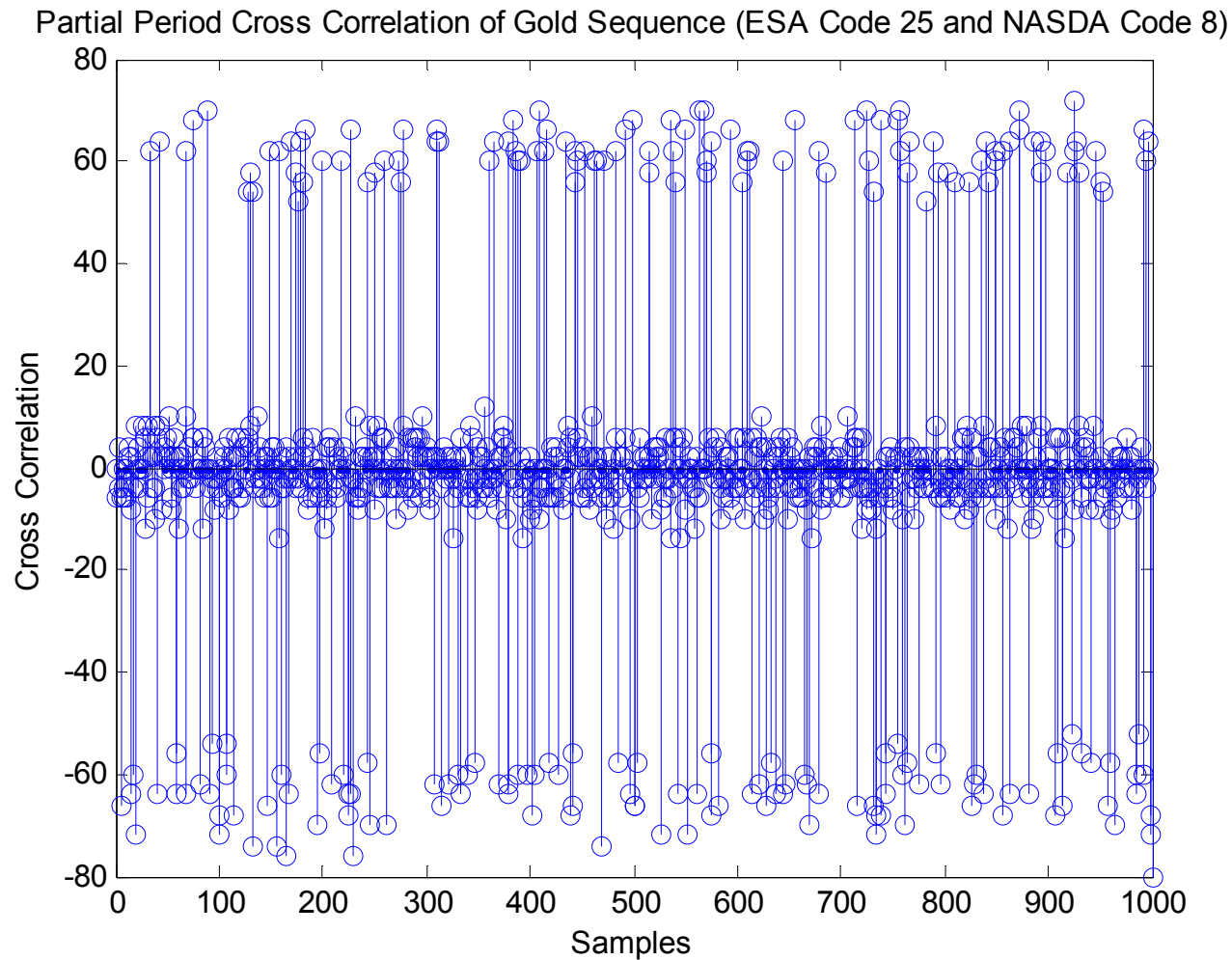


Note: Samples of Cross-Correlation value are taken with randomly generated delay between ATV Code and HTV Code.



# Partial-Period Cross-Correlation of ATV Code and HTV Code

(Partial-Period M=1000 Chips for chip rate 3 Mcps and data rate 3 kbps)

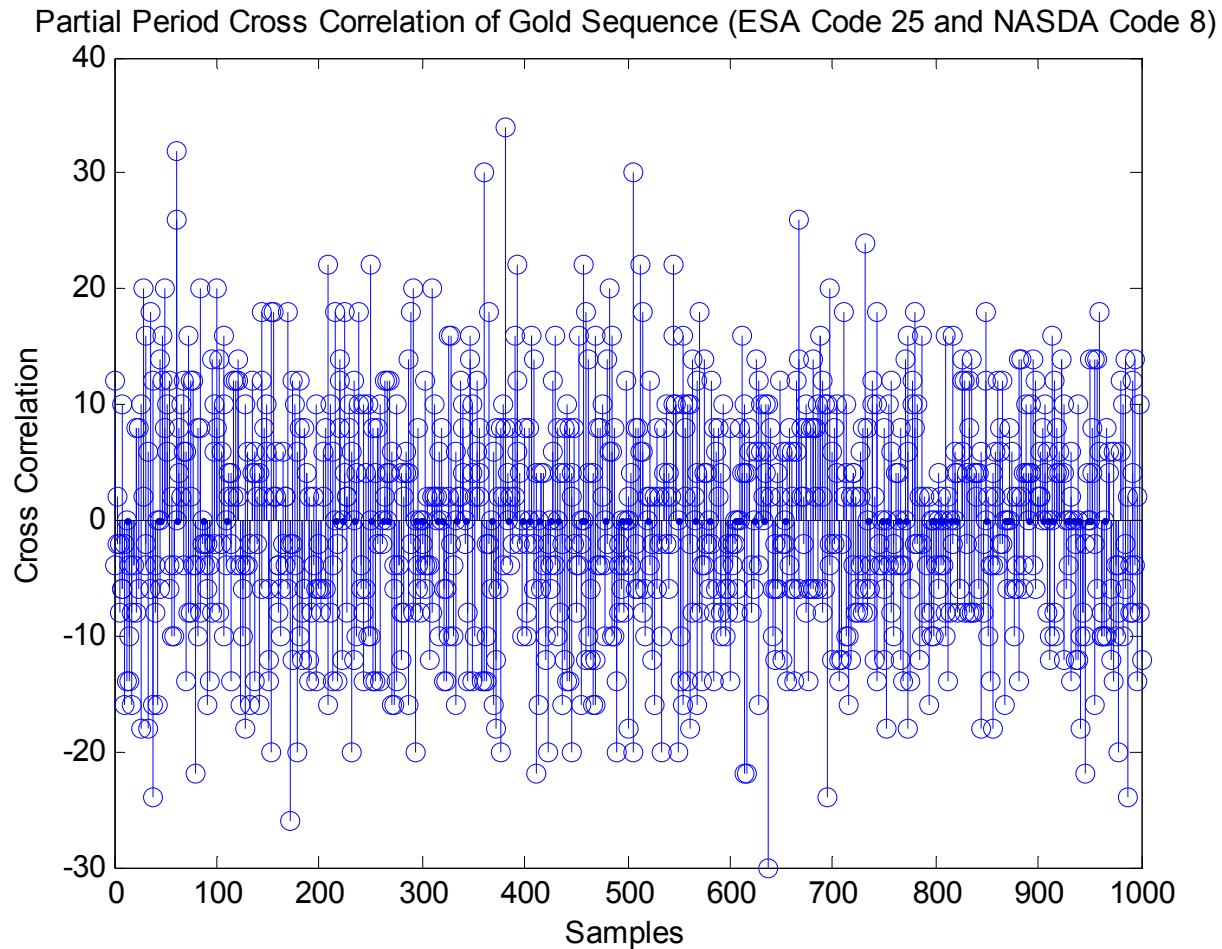






# Partial-Period Cross-Correlation of ATV Code and HTV Code

(Partial-Period M=100 Chips for chip rate 3 Mcps and data rate 30 kbps)



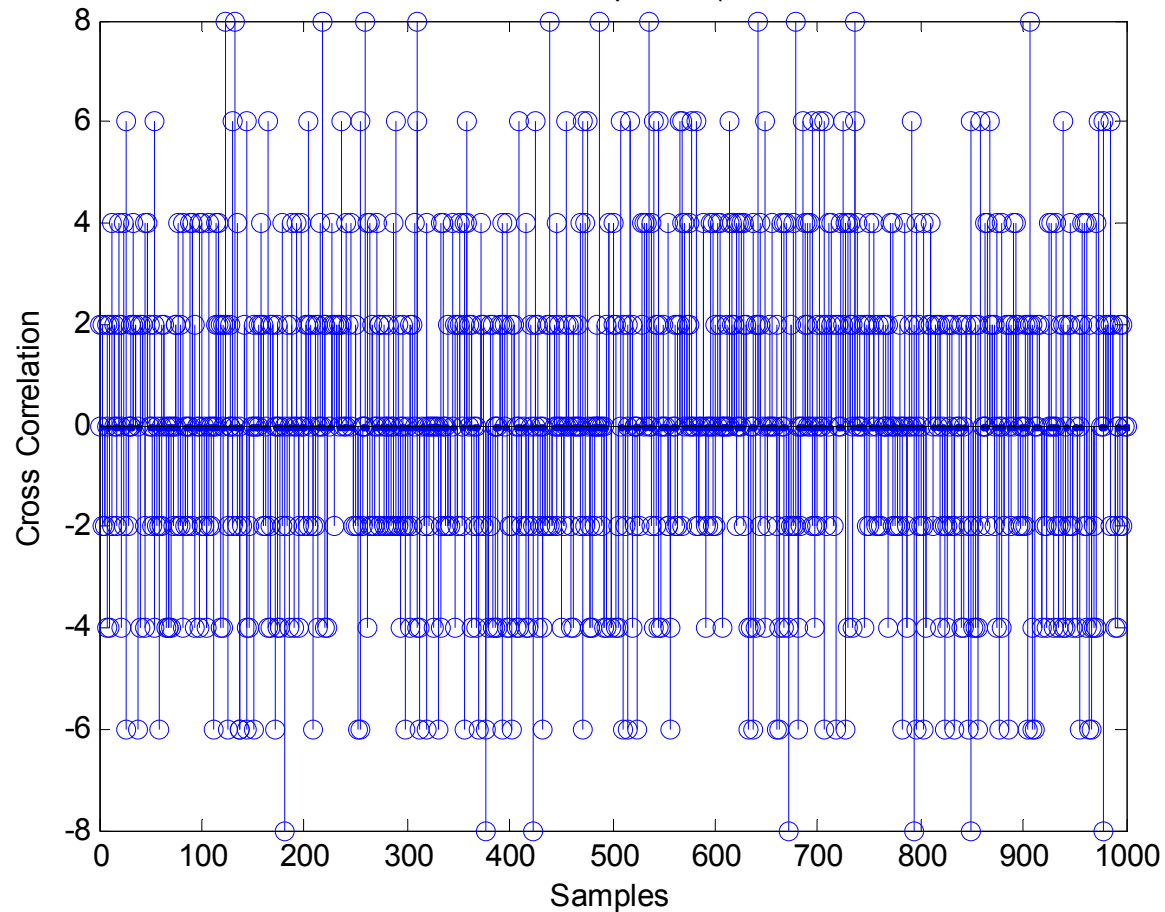


# Partial-Period Cross-Correlation of ATV Code and HTV Code

(Partial-Period M=10 Chips for chip rate 3 Mcps and data rate 300 kbps)



Partial Period Cross Correlation of Gold Sequence (ESA Code 25 and NASDA Code 8)





## Partial-Period Cross-Correlation of ATV Code and HTV Code



- Simulation shows when the data detection integrates over a symbol duration which only spans a portion of a PN sequence due to various data rates, the average partial-period cross-correlation (normalized) can change from 1.6% (designed) up to 25% (high rate 300 kbps).

Partial Period (chips)	Cross-Correlation (abs(max))	Cross-Correlation (abs(mean))	Cross-Correlation (abs(min))	Normalized Cross-Correlation (abs(max))	Normalized Cross-Correlation (abs(mean))	Normalized Cross-Correlation (abs(Min))	Simulation Samples
1023	65	16.6	1	6.4%	1.6%	0.1%	1E4
1000	80	18.5	0	8%	1.9%	0	1E4
100	44	7.9	0	44%	7.9%	0	1E5
10	10	2.5	0	100%	25%	0	1E5



# Summary



- **Part I of this analysis shows that the correlation property of PN sequences is the critical factor which determines the interuser interference performance of the DSSS system.**
- **For non-standard DSSS systems in which PN sequence's period is much larger than one data symbol duration, it is the partial-period cross-correlation that determines the system performance.**
- **This study reveals through an example that a well-designed PN sequence set (e.g. Gold Sequence, in which the cross-correlation for a whole-period is well controlled) may have non-controlled partial-period cross-correlation which could cause severe interuser interference for a DSSS system. The performance degradation due to partial-period cross-correlation will be evaluated in Part II of this analysis in the future.**



## Future Work



- **Since the analytical derivation of performance metric (BER or SNR) based on partial-period cross-correlation is prohibitive, a simulation approach will be employed to evaluate the interuser interference in such cases.**
- **Since the single-user interference is claimed non-Gaussian in most related studies [5-9], an evaluation will be performed and a trade-off study (between Gaussian assumption and non-Gaussian nature) will be conducted to provide an assessment on the proposed RFC analysis approach.**



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